MAC 2313
Summer 2016
Final Exam

Section # ____________ Name ______________________________

UF ID # ____________ Signature ______________________________

A. Sign your scantron on the back at the bottom in ink.

B. In pencil, write and encode on your scantron in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID Number
3) Section Number

C. Under "special codes", code in the test ID number 4, 1.

1 2 3 ● 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. 1) There are 22 (5-point) multiple choice questions, for a total of 110 points (this includes 10-points extra for this exam).
2) The time allowed is 90 minutes.
3) You may write on the test.
4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. **KEEP YOUR SCANTRON COVERED AT ALL TIMES.**

G. When you are finished:

1) Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.
2) Bring your test, scratch paper, and scantron to your proctor to turn them in. Be prepared to show your UF ID card.
3) Answers will be posted in Canvas after the exam.

The Honor Pledge: "On my honor, I have neither given nor received unauthorized aid in doing this exam."

Student's Signature:______________________

Questions 1–22 are worth 5 points each.

1. Let D be a connected and simply connected bounded region in the xy-plane and let ∂D be smooth with a counterclockwise orientation. If $\mathbf{F} = \langle f, g \rangle$ with f and g differentiable, which of the following are true?

   I. $\displaystyle\oint_{\partial D} \mathrm{F} \cdot d\mathbf{r} = \iint_D (g_y - f_x)\, dA$

   II. If $g_y - f_x = 1$, then $\displaystyle\text{Area}(D) = \oint_{\partial D} \mathbf{F} \cdot d\mathbf{r}$

   III. If $\mathbf{F}$ is conservative, then $\text{Area}(D) = 0$

   IV. $\displaystyle\oint_{\partial D} \mathbf{F} \cdot d\mathbf{r} = \oint_{\partial D} f\, dx\, + g\, dy$

   A. I, II, and IV only

   B. I, III, and IV only

   C. IV only

   D. III and IV only

   E. I, II, and III only

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2. Let $\mathbf{F} = \langle y, x \rangle$ and C be any path from (a, b) to (c, d), where (a, b) and (c, d) are distinct points. Find $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{S}$.

   A. 0

   B. $(ab + cd)^2$

   C. $ab + cd$

   D. $ab - cd$

   E. $cd - ab$

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3. Which of the following correctly parametrizes the surface S given as the part of the region bounded by $z = 9$ and $z = 25 - x^2 - y^2$ that satisfies $y \geq 0$?

A. $\mathbf{r}(\theta, z) = \langle \cos(\theta), \sin(\theta), z \rangle,\ 0 \leq \theta \leq \pi,\ 9 \leq z \leq 25$

B. $\mathbf{r}(x, y) = \langle x, y, 25 - x^2 - y^2 \rangle,\ 0 \leq x \leq 4,\ -\sqrt{16 - x^2} \leq q \leq \sqrt{16 - x^2}$

C. $\mathbf{r}(x, y) = \langle x, y, 25 - x^2 - y^2 \rangle,\ -4 \leq x \leq 4,\ 0 \leq y \leq \sqrt{16 - x^2}$

D. $\mathbf{r}(\theta, z) = \langle \cos(\theta), z, \sin(\theta) \rangle,\ 0 \leq \theta \leq \pi,\ 9 \leq z \leq 25$

E. None of the above

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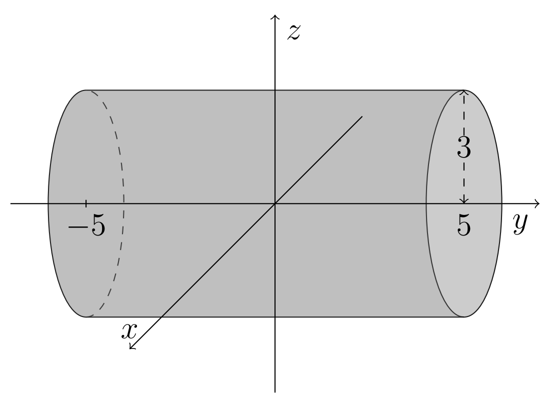
4. Evaluate the flux of the vector field $\mathbf{F} = \langle 3x, x^2 + y, e^{xy} - 2z \rangle$ across the surface of the **closed cylinder** which can be expressed as a union of two surfaces A and B which is given by

$$A = \{(x, y, z) | x^2 + z^2 = 9, -5 \leq y \leq 5\}, \quad B = \{(x, y, z) | x^2 + y^2 \leq 9, y = \pm 5\}$$

The figure is illustrated below



A. 180π

B. 0

C. 1080π

D. 90π

E. $90(\pi - 1)$

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5. Find the circulation $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where C is the triangle with vertices $(-1, 0)$, $(1, 0)$, and $(0, 2)$ oriented counterclockwise, and $\mathbf{F} = \langle e^x, \pi x + \sin(\pi y) \rangle$.

A. -2π
B. 2π
C. 2
D. 0
E. $-\pi$

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6. Find the vector surface integral $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$ for the vector field $\mathbf{F} = \langle e^x, x - y^2, z^3 \rangle$ where S is the part of the ellipsoid $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 + \left(\frac{z}{5}\right)^2 = 1$ where $z \geq 0$ with upward pointing normal.

A. $4(\pi - 1)$
B. 4π
C. $\pi(4e - 1)$
D. 0
E. $\pi(1 - 2e)$

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7. Evaluate $\int_C \sin(x)\ dx + z\cos(y)\ dy + \sin(y)\ dz$ where C is the ellipse $4x^2 + 9y^2 = 36$, oriented clockwise.

A. 4
B. 24
C. 9
D. 0
E. 1

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8. Let $\mathbf{F} = \langle F_1, F_2, F_3 \rangle$ such that the second order partials of the component functions are continuous. Find div(curl($\mathbf{F}$)).

A. $\langle 0, 0, 0 \rangle$
B. -1
C. 0
D. 1
E. None of the above

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9. Let $\mathbf{F}$ be a vector field defined on a simply connected domain in $\mathbb{R}^3$ and ϕ be a scalar function in $\mathbb{R}^3$. Which of the following are true?

    I. $\nabla \cdot (\nabla \times \mathbf{F}) = 0$

    II. $\nabla \times (\nabla \phi) = \mathbf{0}$

    III. If $\mathbf{F}$ is conservative, then $\nabla \cdot \mathbf{F} = 0$

    IV. If $\nabla \times \mathbf{F} = \mathbf{0}$, then $\mathbf{F}$ is conservative

    A. I, II, and IV only

    B. I, III, and IV only

    C. I, II, III, and IV

    D. I and IV only

    E. I and III only

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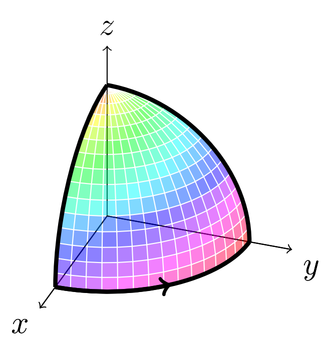
10. Find the line integral of the vector field $\mathbf{F} = \langle x, y, -z \rangle$ on the piecewise smooth path indicated in the figure below. Note that the path is the boundary of the unit sphere $\mathbf{r}(\phi, \theta) = \langle \cos(\theta)\sin(\phi), \sin(\theta)\sin(\phi), \cos(\phi) \rangle$ in the first octant $0 \leq \phi \leq \pi/2, 0 \leq \theta \leq \pi/2$.



A. 0 B. $-\frac{\pi}{2}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{8}$ E. $\frac{3\pi}{8}$

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11. Let $\mathbf{F} = z\hat{k}$ be the vector field (in m/sec) of a fluid in R^3. Calculate the flow rate (in m^3/sec) upward through the part of the plane $z = 2 - 2x - y$ that lies above the first quadrant.

A. $\frac{4}{3}$ m^3/sec
B. $\frac{2}{3}$ m^3/sec
C. $\frac{1}{3}$ m^3/sec
D. 0 m^3/sec
E. $\frac{8}{3}$ m^3/sec

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12. Find the line integral of $\mathbf{F} = \langle 1, y\rangle$ around the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$ where $x, y \geq 0$ oriented clockwise.

A. -18
B. 21
C. 0
D. -15
E. None of the above

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13. $\mathbf{A}$ is a vector field such that $\mathbf{A} = \nabla \times \mathbf{F}$ where $\mathbf{F} = \langle x - y, -2y + x, z\rangle$. Evaluate the flux of $\mathbf{A}$, $\iint_S \mathbf{A} \cdot d\mathbf{S}$ where S is the part of the inverted paraboloid $z = 4 - x^2 - y^2$, $z \geq 0$ oriented such that the z component of the normal vector is positive.

A. 4π
B. -2π
C. 2π
D. π
E. 8π

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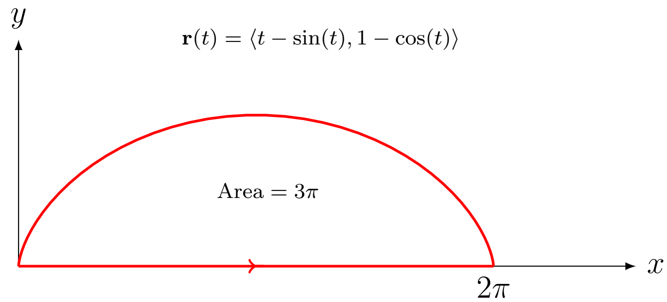
14. Consider the parametrization of a cycloid generated by the unit circle, which has the parametrization

$$\mathbf{r}(t) = \langle t - \sin(t), 1 - \cos(t) \rangle \qquad t \in [0, 2\pi]$$

Together with the line segment from the origin to $(2\pi, 0)$. The curve is illustrated below, note that the **area enclosed by the curve is** 3π.



Find the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ of the vector field $\mathbf{F} = \left\langle x^2, \frac{x}{2} \right\rangle$ on the curve oriented counterclockwise.

A. $\frac{\pi}{2}$ B. $\frac{3\pi}{2}$ C. $\frac{9\pi^2}{2}$ D. $\frac{3\pi^2}{2}$ E. 0

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15. Find the flux of $\nabla \times \mathbf{F}$ over the surface S where $\mathbf{F} = \left\langle x, y, \frac{x}{x^2 + y^2} \right\rangle$ and S is the boundary of the solid cone D of radius 1 and height 2, which is given by

$$D = \left\{ (x, y, z) \,\middle|\, 2\sqrt{x^2 + y^2} \leq z \leq 2 \right\}$$

A. $\frac{1 - 2\pi}{3}$ B. $\frac{4\pi}{3}$ C. $\frac{2\pi}{3}$

D. 0 E. $\sqrt{3}\left(\frac{1}{3} - \frac{\pi}{3}\right)$

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16. Let $\mathbf{F} = \langle 2x + y, x \rangle$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{S}$ where C is any path from $(1, 2)$ to $(5, 7)$.

A. 60 B. 0 C. 57 D. 4 E. 55

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17. Evaluate the integral $\iint_S \sqrt{y^2+z^2}\ dS$ where the surface is given parametrically by $\mathbf{r}(u,v) = \langle v, 2\cos(u), 2\sin(u)\rangle$, $R = \{(u,v) \big| 0 \le u \le 1, 0 \le v \le 2\pi\}$.

A. 2π B. $\frac{\pi}{2}$ C. 8π D. 4π E. 0

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18. Evaluate the integral $\iint_S \mathbf{F}\cdot\hat{n}\ dS$ where $\mathbf{F} = \langle x+y, 0, xz\rangle$, $\mathbf{r}(u,v) = \langle u+1, v, uv\rangle$, and $0 \le u \le 1$, $0 \le v \le 1$ with the surface oriented so that the z component of the unit normal vector is positive.

A. $\frac{5}{12}$ B. $-\frac{2}{3}$ C. $\frac{3}{2}$

D. $-\frac{1}{6}$ E. None of the above

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19. Find the area of the part of the plane $z = x + 2y - 1$ defined on the triangular region of the xy-plane with vertices $(0,0), (1,0),$ and $(0,1)$.

A. 3 B. $\frac{1}{2}$ C. $\sqrt{\frac{1}{2}}$

D. $\frac{\sqrt{6}}{2}$ E. None of the above

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20. Evaluate the flux $\iint_S \mathbf{F}\cdot d\mathbf{S}$ of the vector field $\mathbf{F} = \langle x^2y, 3y, -2xyz\rangle$ Where S is the unit sphere parametrized by $\mathbf{r}(\phi,\theta) = \langle \cos(\theta)\sin(\phi), \sin(\theta)\sin(\phi), \cos(\phi)\rangle$, where $0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi$ with outward pointing normal.

A. 2π B. 4π C. π

D. 0 E. None of the above

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21. (Bonus) If $\mathbf{F}$ is a vector field defined on a simply connected region and $\nabla\cdot\mathbf{F} = 0$, then there exists a vector field $\mathbf{A}$ such that $\nabla\times\mathbf{A} = \mathbf{F}$

A. TRUE B. FALSE

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22. (Bonus) A conservative vector field is a vector field $\mathbf{F} = \langle f, g, h\rangle$ which satisfies the cross partial condition given by

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}, \quad \frac{\partial g}{\partial z} = \frac{\partial h}{\partial y}, \quad \frac{\partial h}{\partial x} = \frac{\partial f}{\partial z}$$

A. TRUE B. FALSE
