MAC 2313
Summer 2016
Exam 3

Section # ____________ Name ____________________________

UF ID # ____________ Signature ____________________________

A. Sign your scantron on the back at the bottom in ink.

B. In pencil, write and encode on your scantron in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID Number
3) Section Number

C. Under "special codes", code in the test ID number 3, 1.

1 2 ● 4 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. 1) There are 14 (5-point) multiple choice questions, plus 4 free response questions for a total of 105 points (this includes 5-points extra for this exam).

2) The time allowed is 90 minutes.
3) You may write on the test.
4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. **KEEP YOUR SCANTRON COVERED AT ALL TIMES.**

G. When you are finished:

1) Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.
2) Bring your test, scratch paper, and scantron to your proctor to turn them in. Be prepared to show your UF ID card.
3) Answers will be posted in Canvas after the exam.

The Honor Pledge: "On my honor, I have neither given nor received unauthorized aid in doing this exam."

Student's Signature:______________________

Questions 1–14 are worth 5 points each.

1. The density function of a certain material is given by $\rho(x, y, z) = \dfrac{1}{\sqrt{x^2 + y^2}}$ What is the mass of an object made with this material having the shape described by

$$D = \{(x, y, z) | x^2 + y^2 \leq 1, 0 \leq z \leq \sqrt{x^2 + y^2}\}.$$

Hint: The mass of an object with a density function $\rho(x, y, z)$ is given by $\iiint_D \rho(x, y, z)\, dV$. Convert to cylindrical coordinates to evaluate the resulting triple integral.

A. π B. 2π C. $\dfrac{\pi}{2}$

D. 4π E. $(2 - \sqrt{2})\pi$

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2. Compute the integral $\displaystyle\int_0^1 \int_2^3 \frac{1}{(x + 4y)^3}\, dx\, dy$

A. $\dfrac{1}{56}$ B. $\dfrac{1}{16}$ C. $\dfrac{3}{8}$ D. $\dfrac{1}{8}$ E. $\dfrac{2}{5}$

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3. Find the volume under the function $f(x, y) = x^2 + y^2$ over the region between the circle $x^2 + y^2 = 1$ and the circle $x^2 + y^2 = 4$ on the xy-plane.

A. $\dfrac{15\pi}{2}$ B. $\dfrac{255\pi}{2}$ C. 8π

D. 4π E. $\dfrac{14\pi}{3}$

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4. Evaluate the integral of $f(x, y, z) = 2y$ over the solid bounded by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.

A. 2 B. 12 C. 4 D. 1 E. 0

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5. Parabolic cylindrical coordinates transforms (x, y, z) to (u, v, z) by the transformation given by $P(u, v, z) = \left(uv, \frac{u^2 - v^2}{2}, z\right)$. Compute the absolute value of the Jacobian $|\text{Jac}(P)|$.

A. $(u + v)^2$ B. $u^2 + v^2$ C. $2uv$

D. $u^2 - v^2$ E. None of the above

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6. Integrate $f(x, y) = x$ over the region bounded by $y = x^2$ and $y = x + 2$.

A. $\frac{9}{4}$ B. 0 C. 2

D. 9 E. None of the above

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7. Evaluate the integral $\iiint_D e^{(x^2+y^2+z^2)^{3/2}}\, dV$ where D is the region above the cone $z = \sqrt{x^2 + y^2}$ and below the hemisphere $z = \sqrt{1 - x^2 - y^2}$ using spherical coordinates.

A. $\frac{\pi}{3}\left(2 - \sqrt{2}\right)(e - 1)$

B. $\frac{2\pi}{3}\left(2 - \sqrt{2}\right)(e - 1)$

C. $\frac{\pi}{3}\left(2 + \sqrt{2}\right)(e - 1)$

D. $\frac{\pi}{3}\left(\sqrt{2} - 2\right)(e - 1)$

E. None of the above

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8. Compute the area of the region in $\mathbb{R}^2$ bounded by the lines $y = 0$, $y = 2$, $y = 2x$, and $y = 2x - 4$.

A. $\frac{5}{4}$ B. 4 C. π D. 2 E. 6

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9. The region R in (ρ, ϕ, θ) space is mapped onto the region S in the Cartesian (x, y, z) space by the standard transformation $T(\rho, \phi, \theta) = (\rho\cos(\theta)\sin(\phi), \rho\sin(\theta)\sin(\phi), \rho\cos(\phi))$. If S is the region between the two spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 9$, what is the volume of the box R?

A. $\frac{104\pi}{3}$

B. $\frac{28\pi}{3}$

C. $4\pi^2$

D. $\frac{32\pi}{3}$

E. $2\pi^2$

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10. Evaluate the double integral $\int_0^1 \int_x^1 xe^{y^3}\, dy\, dx$.

A. $\frac{1}{12}(e-1)$

B. $\frac{1}{6}(e-1)$

C. $\frac{e}{6}$

D. $\frac{e}{6} - 6$

E. 0

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11. Find the extreme values for $f(x, y) = 2x + 4y$ subject to the constraint $g(x, y) = x^2 + y^2 - 5 = 0$.

A. Maximum: 2, Minimum: -1

B. Maximum: 2, Minimum: -2

C. Maximum: 10, Minimum: None

D. Maximum: 4, Minimum: 2

E. Maximum: 10, Minimum: -10

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12. Set up the integral $\iiint_W xz\, dV$, where W is the region bounded by the elliptic cylinder $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and the sphere $x^2 + y^2 + z^2 = 16$ in the first octant.

A. $\int_0^2 \int_0^{\frac{3}{2}\sqrt{4-x^2}} \int_0^{\sqrt{16-x^2-y^2}} xz\, dz\, dy\, dx$

B. $\int_{-2}^2 \int_0^{\frac{3}{2}\sqrt{4-x^2}} \int_0^{\sqrt{16-x^2-y^2}} xz\, dz\, dy\, dx$

C. $\int_0^2 \int_0^{16-x^2} \int_0^{\sqrt{16-x^2-y^2}} xz\, dz\, dy\, dx$

D. $\int_0^2 \int_0^{\frac{2}{3}\sqrt{4-x^2}} \int_0^{\sqrt{16-x^2-y^2}} xz\, dz\, dy\, dx$

E. None of the above

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13. The point $(r, \theta, z) = \left(2, \frac{\pi}{4}, -2\right)$ in cylindrical coordinates can be converted to spherical coordinates (ρ, ϕ, θ) as

A. $\left(2\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{4}\right)$

B. $\left(2, \frac{\pi}{4}, \frac{\pi}{4}\right)$

C. $\left(2\sqrt{2}, \frac{3\pi}{4}, \frac{\pi}{4}\right)$

D. $\left(2, \frac{3\pi}{4}, \frac{\pi}{4}\right)$

E. $\left(2\sqrt{2}, \frac{\pi}{4}, \frac{3\pi}{4}\right)$

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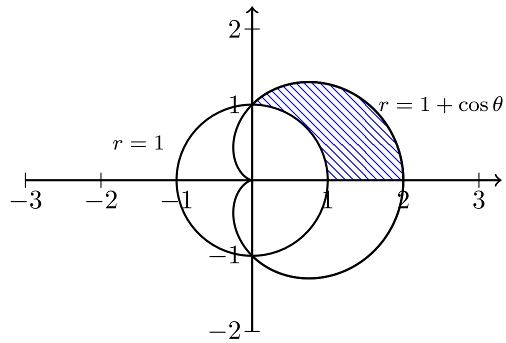
14. Find the area of the shaded region using polar coordinates.



A. $1 - \frac{\pi}{2}$

B. $1 + \frac{\pi}{8}$

C. $\frac{1}{2} + \frac{\pi}{8}$

D. $2 + \frac{\pi}{2}$

E. $1 + \frac{\pi}{2}$

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15. When the integral $I = \int_1^2 \int_0^{\sqrt{2x-x^2}} e^{x^2+y^2}\, dy\, dx$ is converted to a polar integral, you would get:

A. $I = \int_0^{\pi/4} \int_1^{2\cos(\theta)} e^{r^2} r\, dr\, d\theta$

B. $I = \int_0^{\pi/2} \int_0^{2} e^{r^2} r\, dr\, d\theta$

C. $I = \int_0^{\pi/2} \int_{\sec(\theta)}^{2\cos(\theta)} e^{r^2} r\, dr\, d\theta$

D. $I = \int_0^{\pi/4} \int_0^{2\cos(\theta)} e^{r^2} r\, dr\, d\theta$

E. $I = \int_0^{\pi/4} \int_{\sec(\theta)}^{2\cos(\theta)} e^{r^2} r\, dr\, d\theta$

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YOU MUST SHOW ALL WORK TO RECEIVE FULL CREDIT.

Free response questions 1-4 are worth 30 points total.
You must show complete algebraic work to receive credit!

1. [9 Points] Set up, but **do not evaluate** the triple integral $\iiint_R (x^2 + y^2 + z^2)\, dV$ where D is the solid bounded by $z = 6 - x^2 - y^2$, $x^2 + y^2 = 1$ and $z = x^2 + y^2$ in cylindrical coordinates

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2. [6 Points] Let W be the region below the paraboloid $x^2 + y^2 = z - 2$ that lies above the part of the plane $x + y + z = 1$ in the first octant. Express $\int\int\int_W f(x, y, z)dV$ as an iterated integral.

3. [7 points] Sketch the domain D defined by $x + y \leq 12$, $x \geq 4$, $y \geq 4$ and compute $\int\int_D e^{x+y}dA$.

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4. The function $f(x, y) = 3x + 6y$ is defined in the region S which is bounded by $y = x + 1$, $y = x - 1$, $y = -2x + 1$ and $y = -2x + 3$.

   (a) [2 Points] The transformation $T(u, v) = \left(\frac{v-u}{3}, \frac{2u+v}{3}\right)$ transforms the region R on the uv plane onto the given region S on the xy-plane. Describe this region R either in the form of a diagram or as a set.

   (b) [3 Points] Find the Jacobian of this transformation

   (c) [3 Points] Evaluate the integral $\iint_S f(x, y)\, dA$ as a double integral in the variables u and v using change of variables formula.