Midterm Exam 2

Section I: 2 Point Problems - Answer exactly 3 of the following 4

1. Rewrite $y''' - 3y'' - 4y = 2e^{-2x}\cos(3x) + x^5e^{2x}$ as a homogeneous equation using annihilators.

2. Given

$$A = \int_3^{\infty} \left(\frac{\frac{1}{2}t^2 - 2t}{3t-12}\right)\delta(t-6)\,dt, \qquad B = \int_{-\infty}^{3\pi/2} \left(2\cos^2(t) - 1\right)\delta\left(t-\pi\right)\,dt,$$

$$C = \int_{-1}^{3}\left(3e^{t-2} - \frac{t^2}{4} - (t)!\right)\delta(t-2)\,dt,$$

determine $A - B + C$.

3. Given $f(t) = t$ and $g(t) = e^t$, compute $(f * g)(t)$.

4. Compute $\mathscr{L}\left\{te^{2t}\sin(3t) + t^2 * \cos(2t) - 3t^2u(t-1)\right\}$.

Section II: 3 Point Problems - Answer exactly 2 of the following 3

5. Find the general solution to $y''' - y'' + 3y' + 5y = 0$.

6. Use the definition of the Laplace transform to compute $\mathscr{L}\left\{(t-2)^2u(t-2)\right\}$

7. Solve $y'' + 4y = 2u(t-\pi);\ y(0) = 2,\ y'(0) = 0$

Section III: 4 Point Problems - Answer exactly 2 of the following 3

8. Use the Wronskian to determine if $\{e^t, te^t, t\}$ is a set of linearly independent solutions.

9. Solve $y''' - y' = 0;\ y(0) = 0,\ y'(0) = 2,\ y''(0) = 0$ using Laplace transforms.

10. Solve the following system for $x(t)$ and $y(t)$.

$$\begin{cases} x' + y' = x - y, & x(0) = 1 \\ x' - y' = x - y, & y(0) = 0 \end{cases}$$

Bonus Questions:

Bonus # 1: (3 Points - 1 Point per checkpoint)
You are going to use Laplace transforms to determine the appropriate 'Pythagorean' type identity for hyperbolic sine and cosine.

- Part 1: Begin with $\cosh(t) = \dfrac{e^t + e^{-t}}{2}$ and $\sinh(t) = \dfrac{e^t - e^{-t}}{2}$ and compute $\cosh^2(t)$ and $\sinh^2(t)$.
- Part 2: Using your expressions above, compute $\mathscr{L}\left\{\cosh^2(t)\right\}$ and $\mathscr{L}\left\{\sinh^2(t)\right\}$.
- Part 3: Since you know that $\mathscr{L}\{1\} = \dfrac{1}{s}$, use your transforms to decide which identity is appropriate: (try to explain, don't just pick one)

$$\cosh^2(t) + \sinh^2(t) = 1 \qquad \underline{\text{or}} \qquad \cosh^2(t) - \sinh^2(t) = 1$$

Bonus # 2: (1 Point)
Use the definition of the Laplace transform to determine $\mathscr{L}\{u(t-a)\}$.

Bonus # 3: (1 Point)
Fun question - You may have heard the riddle 'You walk 1 mile south, 1 mile east, then 1 mile north. You are at the same place you started. Where are you?' The most common answer to this riddle is the North Pole. However, there is another solution. What is it?

Section I	2	2	2	Section II	3	3	Section III	4	4	Bonus	Total
Score				Score			Score				

Table of Laplace Transforms

$\mathscr{L}\{1\} = \dfrac{1}{s}$	$\mathscr{L}\{e^{at}\} = \dfrac{1}{s-a}$
$\mathscr{L}\{\sin(bt)\} = \dfrac{b}{s^2+b^2}$	$\mathscr{L}\{\cos(bt)\} = \dfrac{s}{s^2+b^2}$
$\mathscr{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$	$\mathscr{L}\{e^{at}f(t)\} = F(s-a)$
$\mathscr{L}\{y'\} = sF(s) - y(0)$	$\mathscr{L}\{y''\} = s^2F(s) - sy(0) - y'(0)$
$\mathscr{L}\left\{y^{(n)}\right\} = s^nF(s) - s^{n-1}y(0) - s^{n-2}y'(0) - \cdots - y^{(n-1)}(0)$	$\mathscr{L}\{t^nf(t)\} = (-1)^n\dfrac{d^n}{ds^n}(F(s))$
$\mathscr{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$	$\mathscr{L}\{f(t)*g(t)\} = F(s)\cdot G(s)$