Midterm Exam 1

Section I: 2 Point Problems - Answer exactly 3 of the following 4

1. Solve $\dfrac{dy}{dx} = \dfrac{e^x + e^x y^2}{e^x + 1}$ explicitly for y.

2. Given that $\ddot{y} + 5\dot{y} - 6 = 2e^t$ has general solution $y(t) = c_1 e^{-6t} + c_2 e^t + \frac{2}{7} t e^t$, find the solution that satisfies $y(1) = 0, \dot{y}(1) = \frac{16}{7} e$.

3. Find the special integration factor that will make $(x^2)dx + \frac{1}{3}(x^3 y - x^3 \sin(y))dy = 0$ exact.

4. Find the general solution to $\dfrac{dy}{dx} = \frac{y}{x} + x \csc\left(\frac{y}{x} + 1\right)$. (Hint: Use homogeneous methods)

Section II: 3 Point Problems - Answer exactly 2 of the following 3

5. Find the general solution to $\dfrac{dy}{dx} - 2xy = -4x^3 y^3$.

6. Solve the IVP $(\sin(y) - y\sin(x))dx + (x\cos(y) + \cos(x))dy = 0 \qquad y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$.
(Hint: Remember general solutions have the form $F(x, y) = C$)

7. Find a (nonzero) particular solution to the nonhomogeneous equation $\ddot{y} - 4\dot{y} + 13y = 2te^{2t}$.

Section III: 4 Point Problems - Answer exactly 2 of the following 3

8. Find the general solution to the nonhomogeneous equation $2\ddot{y} - 3\dot{y} - 2y = 2\sin(2t)$.

9. Find a particular solution to the nonhomogeneous equation $\ddot{y} + 4\dot{y} + 4y = e^{-2t}\log(t)$.

10. A nitric acid solution flows into a tank at a rate of 2L/min. The tank initially has 10L of a 20% nitric acid solution. The solution is well stirred and leaves the tank at 4L/min. If the solution entering the tank is 50% nitric acid, determine the volume of nitric acid in the tank after t minutes.

Bonus Questions:

Bonus # 1: (3 Points - 1 Point per checkpoint)
You are going to illustrate the derivation of the integration factor for first order linear differential equations.

- Part 1: Begin with $a_1(x)\dfrac{dy}{dx} + a_0(x)y = b(x)$ and write it in standard form - make sure to define what your new functions of x are, i.e. $P \,\&\, Q$.

- Part 2: Multiply your standard form by $\mu(x)$ and derive the equation needed to solve for $\mu(x)$. (Hint: expand $\frac{d}{dx}(\mu(x)y)$ and match it with your multiplied standard form - you should get a separable equation)

- Part 3: Solve the separable equation you obtained to find the definition of $\mu(x)$ you know and love and write down what the solution $y(x)$ will look like.

Bonus # 2: (1 Point)
Explain the superposition principle for nonhomogeneous second order linear differential equations with constant coefficients.

Bonus # 3: (1 Point)
Fun question - What is the shape of the Pac-Man universe?

Section I	2	2	2	Section II	3	3	Section III	4	4	Bonus	Total
Score				Score			Score				